Sector-based Distributed Scheduling Strategy in Air Traffic Flow Management

Yicheng Zhang * Qing Li ** Rong Su ***

* School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore 639798. (e-mail: yzhang088@e.ntu.edu.sg)
** Air Traffic Management Research Institute (ATMRI) at Nanyang Technological University, Singapore 639798. (e-mail: liqing@ntu.edu.sg)
*** School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore 639798. (e-mail: rsu@ntu.edu.sg)

Abstract: Air traffic flow scheduling is an important component in air traffic control system and has significant effects on the safety and efficiency of air transportation. In this paper, we propose a sector-based distributed air traffic flow scheduling strategy to minimize the airport arrival delay. The scheduling problem is formulated based on a simplified air traffic system model consisting of sectors, air-routes, waypoints and airports. A cell transmission flow dynamic model is adopted to describe the system dynamics and some safety-based constraints such as the sequence of en-routed aircraft and the limitations of cruising speeds are considered in the system model. To overcome the computational complexity involved in the centralized structure, we propose a sector-based distributed approach based on Lagrangian relaxation. Experimental results demonstrate the effectiveness of the proposed scheduling strategies.

Keywords: air traffic flow management, Lagrangian relaxation, en-route air traffic system, distributed optimization

1. INTRODUCTION

According to a news release dated back to December 10, 2013 by the International Air Transport Association (IATA), airlines expect to see 31% increase of passengers by 2017 with a compound annual growth rate (CAGR) of 5.4%. Asia-Pacific passenger traffic is forecast to grow at 5.7% CAGR. Traffic within the Asia-Pacific region will represent 31.7% of global passengers in 2017, up from 28.2% in 2012. As one of the hubs in the Asia-Pacific region, Singapore saw 53.7 million passengers passed through the airport in 2013. How to undertake effective air traffic flow management to reduce air traffic delay, which has cost billions of dollars to airlines, Ball et al. (2010), is one of the major challenges faced by the aviation industry.

Many relevant researches proposed in literatures try to solve the air traffic flow management problem. A comparison of dynamic models adopted in air traffic flow management is provided in Sridhar and Menon (2005). Traditional models are based on simulating the trajectories of individual aircraft. However, this approach does not offer any insights into the dynamics of the traffic flow. The aggregate traffic model and Eulerian model are provided to describe the behaviour of air traffic flow. The aggregation approach provides a lower order, fixed-resolution model of the airspace, while the Eulerian approach provides a flexible resolution model, Sridhar et al. (2006). The aggregate approach is only able to control the number of aircraft in the center but not the aircraft in each link. Thus, it is not suitable for the en-route air traffic flow management problem. A multicommodity Eulerian-Lagrangian large-capacity cell transmission model for en-route air traffic is provided in Sun and Bayen (2008), which utilizes a standard multi-commodity flow model equipped with origin-destination (OD) pairs to avoid difficulties in describing flow merging and diverging.

In this paper, we first propose a system formulation for air traffic flow management in quadratic integer programming form. Although a quadratic programming problem can be solved efficiently compared with some other optimization formulations, for a large-scale system we may still face a challenge of high computational complexity due to an extremely large number of decision variables and constraints, even though it is polynomial-time, Megiddo (1987). For this reason, we develop a distributed scheduling algorithm to solve this quadratic integer programming problem, aiming for a good trade-off between the quality of scheduling and the computational complexity.

This paper addresses the problem of air traffic flow management, which covers flight scheduling among different airports via relevant airspace, to minimize the airport arrival delay in air traffic system. In this paper, the air traffic flow scheduling problem is formulated based on a flow model, aiming to reduce the total arrival delay time in the system. A simplified air traffic system modelling method is provided and a sector-based distributed air traffic flow scheduling strategy is proposed based on this system model. We describe an air traffic system contain...
ing airports, waypoints and air-routes with loops. The dynamics are modelled by a flow dynamic model similar to cell transmission models, Daganzo (1994), Daganzo (1995). Some constraints, which have been ignored in the literature but important to air traffic controller and system safety, such as the lower and upper limitation of the cruising speeds are considered in this model. The air traffic flow scheduling problem is formulated as a quadratic integer programming problem with a set of linear constraints. The centralized air traffic flow scheduling problem can be solved by using some commercial optimization solvers such as optimization toolbox in MATLAB or CPLEX, ILOG (2014). However, the computational complexity issue occurs during this procedure as a large amount of computational time and memory sources are needed. To overcome this issue involved in centralized structure, we propose a sector-based distributed scheduling strategy based on the Lagrangian relaxation, Bertsekas (1999) and subgradient method, Boyd et al. (2003), Zhao et al. (1999), Nedić et al. (2001). Numerical experiments are given to show the effectiveness of both centralized and distributed approaches. Our contributions include, firstly, a centralized problem formulation to solve the air traffic scheduling problem based on quadratic integer programming form, and secondly a distributed approach to enhance the efficiency of computation.

The rest of the paper is organized as follows. An air traffic system model is proposed in Section 2. The centralized air traffic flow scheduling problem formulation is presented in Section 3 and the sector-based distributed air traffic flow scheduling strategy is proposed in Section 4. Some experimental results are provided in Section 5. Conclusions are drawn in Section 6.

2. SYSTEM MODEL DESCRIPTION

An air traffic system includes airports and the air-route grid connected to them. The airport include departure links, arrival links and some holding links, while the air-route grid consists of waypoints (or control points) and air-routes between each two waypoints. A waypoint is a reference point in physical space used for purposes of navigation, i.e., the aircraft may communicate with the sector control centre when it goes through the waypoint to notice the sector controllers about its location. There are many different types of waypoints in the real world based on different usages from ICAO standards, e.g., DVOR/DME and NDB. In the proposed system model, all the waypoints are considered as the same as their types will not affect the modelling issues. For each individual aircraft, it departs from the original airport, goes through the air traffic grid via some waypoints and finally reaches the destination. In our model, we consider the air traffic flow rather than individual aircraft movements. The air traffic flows are the aggregated aircraft movements from the original airport to the arrival airport via some en-routed air ways.

A basic system module proposed in this paper is shown in Figure 1. It is a simplified version of the air-routes and airports in real air traffic system. The air traffic grid is modelled by air routes and way points, while the airport is modelled by departure point, arrival point and holding pattern. All air-routes are uni-directed with predefined sources and sinks. However, if a bi-directed network to simulate an air traffic system with different flight levels is needed, this system architecture is sufficient to model it by adding paralleled opposite uni-directed air-routes.

3. AIR TRAFFIC FLOW MANAGEMENT PROBLEM

In this section, we will introduce a formulation of a centralized air traffic flow scheduling problem based on quadratic integer programming.

3.1 Notations

Some notations for air traffic flow management problem formulation are listed below. A directed graph \( G = (V, E) \) is adopted to describe the en-route air traffic system. \( G \) denotes the directed graph, while \( V \) and \( E \) means the vertices and edges in this graph, respectively. The directed graph is to describe the air traffic system, with vertices and edges inside, meaning for the waypoints or airports and air-routes, respectively. The vertices in the graph, \( v \in V \), denote the waypoints in the airspace or airports. \( W \subseteq V \) means the collection of all waypoints, while \( A \subseteq V \) means the collection of all concerned airports. The item in \( A \) means airport, denoted as \( a \in A \). Each airport \( a \in A \) consists of a departure point, an arrival point, a departure air-route and an arrival air-route linked to the air traffic grid and a holding link for air-borne holding, \( i := (v, v') \in E \subseteq V \times V \) denotes the air-route departing from node \( v \) to \( v' \). The node can be an airport or another waypoint and the air-route is uni-directed. Some parameters for air-route include the length, the capacity and the connections among each other.

Some parameters for the flights are listed as follows. \( \phi \in \mathcal{F} \) denotes the set of aircraft types. Different type of aircraft has different flight parameters, e.g., the lower bound and upper bound of cruising speeds.

- \( S(\phi) \) – The lower speed limit of aircraft with type \( \phi \).
- \( \overline{S}(\phi) \) – The upper speed limit of aircraft with type \( \phi \).
- \( P := (a, a') \in \mathcal{P} \) denotes the commodities in the air traffic system are defined as the origin-destination pairs.
of airports, \(a, a' \in A\) mean the airports in this system. So if there are \(n_A\) airports and the flights between all airports are flown, then there are \(n_A(n_A - 1)\) commodities in the system. \(P_0\) denotes the original commodity, which means that the aircraft departs from a specific original airport \(a\) to one specific destination airport \(a'\). Here, \(P \in \mathcal{P}\) means an arbitrary commodity in this system. \(P = P_0\) means that the aircraft finishes its flying from the predefined original airport to the destination airport, i.e., with the original commodity, while \(P \neq P_0\) means the aircraft flies with a commodity different with the original one. \(N_P^i(\phi, t)\) – The link volume or the number of aircraft with type \(\phi\) in air-route \(i\) with commodity \(P\) at time interval \(t\). \(f_{\text{in}}^P(\phi, t)\) – The flow rate of aircraft with type \(\phi\) in air-route \(i\) towards air-route \(j\) with commodity \(P\) at time interval \(t\).

3.2 Constraints

All constraints we consider in ATFSP include the network dynamics constraints, the link capacity constraints, the flow rate limits constraints and some integer constraints for variables.

Network dynamics constraints The network dynamics constraints are to describe the relationship between the air traffic flow rates and the air-route volumes, and the cell transmission model is adopted in this formulation,

\[
\forall t \in H_p, i \in E, \phi \in \mathcal{F}, P \in \mathcal{P}, \quad N_P^i(\phi, t + 1) = N_P^i(\phi, t) + \left[ f_{\text{in}}^P(\phi, t) - f_{\text{out}}^P(\phi, t) \right] \Delta
\]

where \(N_P^i(\phi, t)\) and \(N_P^i(\phi, t + 1)\) are the link volume of aircraft with type \(\phi\) in air-route \(i\) with commodity \(P\) at time interval \(t\) and \(t + 1\), respectively. \(f_{\text{in}}^P(\phi, t)\) and \(f_{\text{out}}^P(\phi, t)\) mean the total incoming and outgoing flow rate of aircraft with type \(\phi\) in air-route \(i\) with commodity \(P\) at time interval \(t\).

\[
\begin{align*}
&f_{\text{in}}^P(\phi, t) = \sum_{j \in U_i} f_{ij}^P(\phi, t) \\
&f_{\text{out}}^P(\phi, t) = \sum_{k \in D_i} f_{ki}^P(\phi, t)
\end{align*}
\]

where \(\sum_{j \in U_i} f_{ij}^P(\phi, t)\) means the incoming flow rate of aircraft with type \(\phi\) from all air-routes \(j \in U_i\) in the upper stream link set with commodity \(P\) at time interval \(t\), and \(\sum_{k \in D_i} f_{ki}^P(\phi, t)\) means the outgoing flow rate of aircraft with type \(\phi\) to all air-routes \(k \in U_i\) in the downstream link set with commodity \(P\) at time interval \(t\).

Minimal horizontal separation constraint and link capacity constraint Radar separation is applied by a controller observing that the radar returns from the two aircraft are a certain minimum horizontal distance away from each other, as observed on a suitably calibrated radar system. From this horizontal separation constraint, the air-route capacity can be defined as follows.

\[
C_i = \frac{L_i}{m_{\text{sep}}}
\]

where \(C_i\) is the capacity of air-route \(i\), \(L_i\) is the length of this air-route \(i\), and the minimal separation distance is \(m_{\text{sep}} = 5\text{NM}\). Moreover, as the capacity of air-route may affect by the air or weather condition, it should not be a fixed value but a function based on time \(t\), denoted as \(C_i(t)\).

\[
C_i(t) = \theta(t) \frac{L_i}{m_{\text{sep}}}
\]

where \(\theta(t)\) is a coefficient related to air or weather condition. The number of all types of aircraft in air-route \(i\) should not be greater than the air-route capacity.

\[
\forall t \in H_p, \sum_{P \in \mathcal{P}} \sum_{\phi \in \mathcal{F}} N_P^i(\phi, t) \leq C_i(t)
\]

Flow rate limits constraints With the constraints of the cruising speeds for different types of aircraft, the flow rate should be in a range related to them, i.e., the air-route flow rate is bounded by the product of air-route density and speed limits.

\[
\forall t \in H_p, (i, j) \in E, \phi \in \mathcal{F}, P \in \mathcal{P}, \quad \frac{N_P^i(\phi, t)}{L_i} S(\phi) \leq \sum_{j \in D_i} f_{ij}^P(\phi, t) \leq \frac{N_P^i(\phi, t)}{L_i} S(\phi)
\]

where \(j \in D_i\) means all the air-routes in the downstream set of air-route \(i\), and \(\frac{N_P^i(\phi, t)}{L_i}\) is the air-route capacity.

Integer constraints for variables Some integer constraints for air-route volumes and flow rates are listed here.

\[
\forall t \in H_p, i, j \in E, \phi \in \mathcal{F}, P \in \mathcal{P}, \quad N_P^i(\phi, t) \in \mathbb{Z} \quad (7a)
\]

\[
f_{ij}^P(\phi, t) \in \mathbb{Z} \quad (7b)
\]

\[
C_i(t) \in \mathbb{Z} \quad (7c)
\]

All the air-route volumes, flow rates and link capacities in this system should be integer.

3.3 Objective function

Our objectives for the ATFSP are to minimize the total arrival delay as well as to take originally planned destination into consideration. Based on the aforementioned functions, the objective function can be formulated as follows,

\[
\min \sum_{t \in H_p} \sum_{\phi \in \mathcal{F}} \{ \sum_{j \in U_i} \left[ f_{ij}^{P_0}(\phi, t) - r_{ij}^{P_0}(\phi, t) \right]^2 + \sum_{P \neq P_0} M f_{ij}^P(\phi, t) \} \]

where \(M\) is a very large positive constant, denoting the extremely high penalty on landing aircraft to an airport different from their originally planned destinations, i.e., \(P \neq P_0\), the aircraft takes a different commodity compared with the original one, as described by the second term in the cost function. The first term in the cost function denotes the total arrival delay in all links. Thus the ATFSP can be formulated as an integer quadratic programming problem, with a quadratic objective function, a linear constraint set and an integer constraint set. The integer quadratic programming problem formulation is as follows,
minimize\[\sum_{t \in H_p} \sum_{\phi \in \phi} \left\{ \sum_{j \in U_i} \left[ f_{ij}^{P}(\phi, t) - r_{ij}^{P}(\phi, t) \right]^2 + \sum_{P \neq P_o} M f_{ij}^{P}(\phi, t) \right\} \]
subject to
\[N_{ij}^{P}(\phi, t + 1) = N_{ij}^{P}(\phi, t) + f_{in}^{P}(\phi, t) - f_{out}^{P}(\phi, t) \Delta \]
\[f_{in}^{P}(\phi, t) = \sum_{j \in U_i} f_{ik}^{P}(\phi, t) \]
\[f_{out}^{P}(\phi, t) = \sum_{k \in D_t} f_{kk}^{P}(\phi, t) \]
\[C_i(t) = \theta(t) \frac{L_i}{m_{sep}} \]
\[\sum_{P \in P} \sum_{\phi \in \phi} N_{ij}^{P}(\phi, t) \leq C_i(t) \]
\[\frac{N_{ij}^{P}(\phi, t)}{L_i} S(\phi) \leq \sum_{j \in D_t} f_{ij}^{P}(\phi, t) \leq \frac{N_{ij}^{P}(\phi, t)}{L_i} S(\phi) \]
\[N_{ij}^{P}(\phi, t) \in \mathbb{Z} \]
\[f_{ij}^{P}(\phi, t) \in \mathbb{Z} \]
\[C_i(t) \in \mathbb{Z} \]

Note that here the integer constraints can be relaxed thus the problem becomes a quadratic programming problem rather than quadratic integer programming problem and the benefits of convexity in computation are obtained. Some heuristic approaches are required to obtain the integer solutions for the quadratic-relaxed problem, which will be shown in our further work.

4. SECTOR-BASED DISTRIBUTED AIR TRAFFIC FLOW SCHEDULING STRATEGY

In order to achieve a real time solution, a sector-based distributed air traffic flow scheduling strategy is proposed to solve this quadratic integer programming. We divide the whole airspace based on sectors and use Lagrangian relaxation to relax the boundary constraints. The Lagrangian dual problem is solved by subgradient method and with a parallel manner.

4.1 Distributed System Model

Given a large-scale air traffic system, we can partition it into sub-airspaces based on sector division. Each airport or waypoint only belongs to one sub-airspace, and so does each air-route, except for a few inside the network which are shared by two sub-airspaces. To formally describe the concept of sector-based sub-airspaces, we introduce some terminologies to interpret the distributed problem.

We consider the network as a directed graph \( G = (V, E) \), where the vertex set is \( V = W \cup A \cup \{ext\} \) with \( ext \) denoting the external of the network, and the edge set is \( E \subseteq V \times V - \{(ext, ext)\} \) denoting the set of all one-way air-routes, i.e., each one-way air-route \((v, v') \in E\) represents an air traffic flow either from one intersection \(v\) to another waypoint \(v'\), or from the external source \(v = ext\) to an waypoint \(v'\) (which is an incoming boundary link), or from an waypoint \(v\) to the external source \(v' = ext\) (which is an outgoing boundary link).

Let \( S \) be a partition of the waypoint set and airport set \( W \cup A \), i.e., each waypoint set and airport set which belongs to a single sector and let \( L(S) \) denote all air-routes belong to \( S \). We now make the following modification to the network graph \( G'\): for each link \((v, v') \in E\) with \( v \in S \in S', v' \in S' \in S \neq S', \) we add a node \(b_{v,v'}\) to \( V \), which represents a boundary of \( S \) and \( S' \), and replace \((v, v') \in E\) by two new edges \((v, b_{v,v'})\) and \((b_{v,v'}, v')\); which denotes two disjoint air-route segments, whose union is the original link \((v, v')\). After the modification, let \( B \) be the collection of all such boundary nodes, and \( E' \) be the new edge set. Then the new network graph is \( G' = (V' = V \cup B, E') \), where \( E' \subseteq ((\text{ext}) \cup W \cup A \cup B) \times ((\text{ext}) \cup W \cup A \cup B) - ((\text{ext}, \text{ext})) \cup B \times B \). For any two different sectors, they can only share some boundary waypoints in \( B \).

4.2 Lagrangian Relaxation

Suppose the whole airspace has \( n_k \) separated sectors \( S_k \in S, k = 1, \ldots, n_k \). The boundary constraints are the consistent constraints for the air traffic flow on the boundary air-routes, i.e., at any time interval \( t \in H_p\), the incoming flow should be equal to the outgoing flow on the boundary air-routes, \( f_{in}^{(v,b_{v,v'})} = f_{out}^{(v,b_{v,v'})} \); where \((v, b_{v,v'}) \in L(S)\) and \((b_{v,v'}, v') \in L(S')\) denote the boundary air-route segments in two adjacent sectors. Let \( L(S) := \cup_{S \in S} \cup_{S} \) be the set of all links. Based on the previous terminologies, the proposed ATFSP can be written in the formulation as follows,

\[\text{minimize} \sum_{S \in \delta} J(S)\]
subject to \(\Phi(S)\)
\[\forall (v, b_{v,v'}), (b_{v,v'}, v') \in L(S), f_{in}^{(v,b_{v,v'})} = f_{out}^{(v,b_{v,v'})}\]

where the sector objective function \( J(S) \) is defined as follows,

\[J(S) = \sum_{S' \in S} \sum_{t \in H_p} \sum_{\phi \in \phi} \left\{ \sum_{j \in D_t} \sum_{\phi} \left[ f_{ij}^{P}(\phi, t) - r_{ij}^{P}(\phi, t) \right]^2 + \sum_{P \neq P_o} M f_{ij}^{P}(\phi, t) \right\}\]

and \(\Phi(S)\) denotes the constraint set mentioned in last section. By adopting Lagrangian relaxation, we can remove the boundary constraints in formulation (10) and obtain the following Lagrangian dual problem.

\[\max_{\lambda_{b_{v,v'} \geq 0}, b_{v,v'} \in B} \min_{S \in \delta} J(S)\]
\[\text{subject to } \Phi(S)\]

Let \(\lambda\) be the vector consisting of all \(\lambda_{b_{v,v'}}, b_{v,v'} \in B\) and define \(H(\lambda, S)\) as follows:
\[ H(\lambda, S) : \]
\[
\min J(S) - \sum_{b_i, v' \in B: v' \in S} \lambda_{b_i, v'} f^{in}_{b_i, v'} + \sum_{b_i, v' \in B: v' \in S} \lambda_{b_i, v'} f^{out}_{b_i, v'} \tag{13a}
\]
subject to \( \Phi(S) \) \tag{13b}

Then the Lagrangian dual problem (13a)-(13b) can be rewritten in the following separable form,
\[
\max_{\lambda \geq 0} \sum_{S \in S} H(\lambda, S) \tag{14}
\]

Problem (14) can be solved via a distributed subgradient method.

5. EXPERIMENTAL RESULTS

A large-scale example for this system model is shown in Fig 2. We use an architecture similar to Manhattan model which is frequently used in urban transportation system to model the air traffic grid. The airports are connected to the waypoints at the edge of air traffic grid. Note that we use a bi-directed link to denote the departure and arrival links connected to the airports.

Fig. 2. A 8-8 air traffic system

5.1 Experimental results of centralized ATFSP

The optimization problem is solved by CPLEX based on MATLAB on a PC with an Intel Core(TM) i7-4770 @3.40GHz CPU and RAM 8GB. The sampling time is chosen as 20 minutes and the prediction horizon is chosen as \( H_p = 5, 6, 8, 10 \), respectively. Some parameters about the problem scales are shown in Table 1. We choose two air traffic grid scales, \( n_h = n_v = 4 \) or 8 and the numbers of decision variables, inequality constraints and equality constraints are shown in this table. For example, the ATFSP for an 8-to-8 air traffic system with 16 airports (as shown in Figure 2) consists of 384880 decision variables, 880 inequality constraints and 96880 equality constraints. A problem with such scale is hard to be solved via centralized way because of the limitation of computational complexity. As the results shown in Table 1, we can only obtain the acceptable processing times with some relatively small scale air traffic systems via the centralized air traffic flow scheduling strategy. The memory consumptions in MATLAB for the 4-to-4 air traffic system are shown in Table 2, which indicate that the space complexity is a limitation for this problem as well as the time complexity. To solve the space and time complexity caused by the centralized structure, a sector-based distributed ATFSP formulation is proposed in the following section.

Table 1. Problem Scales and Experimental Results

<table>
<thead>
<tr>
<th>( n_h = n_v )</th>
<th>( H_p )</th>
<th>Decision Variables</th>
<th>Inequalities</th>
<th>Equalities</th>
<th>Processing Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>27169</td>
<td>280</td>
<td>7896</td>
<td>24.6s</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>32592</td>
<td>336</td>
<td>8400</td>
<td>82.4s</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>43456</td>
<td>448</td>
<td>9408</td>
<td>225.9s</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>54320</td>
<td>560</td>
<td>10446</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>384880</td>
<td>880</td>
<td>96880</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>461856</td>
<td>1056</td>
<td>100896</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>615808</td>
<td>1408</td>
<td>108928</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>769760</td>
<td>1760</td>
<td>116960</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2. Memory consumption in MATLAB

<table>
<thead>
<tr>
<th>( n_h = n_v )</th>
<th>( H_p )</th>
<th>Memory Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>1738244Kb</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>2228764Kb</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>3352676Kb</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>out of memory</td>
</tr>
</tbody>
</table>

5.2 Experimental results of distributed ATFSP

The proposed sector-based distributed air traffic flow scheduling strategy is tested under the same PC configuration mentioned before. We apply the aforementioned distributed approach to the air traffic system shown in Fig 2. We consider each sector consisting of \( 4 \times 4 \) waypoints. Thus for this air network with \( 8 \times 8 \) waypoints, we have four sectors to consider.

Table 3. Details of an 8-to-8 air traffic system

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_h = n_v )</td>
<td>8</td>
</tr>
<tr>
<td>Number of airports</td>
<td>16</td>
</tr>
<tr>
<td>Number of waypoints</td>
<td>80</td>
</tr>
<tr>
<td>Number of air-routes</td>
<td>176</td>
</tr>
<tr>
<td>Number of commodities</td>
<td>240</td>
</tr>
<tr>
<td>Length of Sampling time</td>
<td>20 mins</td>
</tr>
<tr>
<td>Length of prediction time</td>
<td>100 mins</td>
</tr>
<tr>
<td>Number of decision variables</td>
<td>384880</td>
</tr>
</tbody>
</table>

This problem cannot be solved with a centralized structure as the needed memory is out of the limitation of PC. With distributed approach, this problem can be solved around 490 seconds (about 8 minutes).
5.3 A Case Study – A simplified air traffic system model for South-East Asian Region

We generate a simplified air traffic system based on the en-routed map provided by ICAO for south-east Asian region. 10 airports with 26 waypoints are selected to build the system model and a brief map for this region is shown in Fig 3. The airports are located in Singapore, Kuala Lumpur, Penang, Phuket, Bangkok, Ho Chi Minh City, Kota Kinabalu, Kuching, Jakarta and Surbaya, respectively. This area is one of the world’s busiest air traffic region and 4 of these 10 airports are ranking in the top 30 busiest airports of the world.

All the flights in this case study are departed from Changi Airport in Singapore. Thus a total of 9 origin-destination pairs of airports are considered in this system. As shown in website, Changi Airport handles about 6,500 flights every week and over 53 million passengers a year. Due to the busy traffic condition, we choose a relatively short sampling time, 6 minutes. Thus in our model, about 5 flights depart from Changi Airport every 6 minutes. Notice that in real world there will be different flight levels existing in the air-routes to satisfy the preferred flight levels of different aircraft types. In this case study, we suppose that all the aircrafts in this system cruise on the same flight level. We test the proposed approach in a time period of 4 hours and 40 sampling points are included in this case. The problem can be solved around 20 minutes.

Fig. 3. A brief en-route air traffic system map for South-East Asian region

6. CONCLUSIONS

In this paper, we have proposed a realistic modelling method for en-route air traffic system and an approach to minimize the arrival delay both in centralized and distributed structure. We extend an Eulerian-Lagrangian flow model to describe the network dynamics with possibly different aircraft types, and formulate a quadratic integer programming problem for delay reduction under constraints of limited link capacities along with possibility of flight rerouting and diversion. We show that such an Eulerian-Lagrangian dynamic model and the quadratic integer programming optimization formulation can render sufficiently rich expressiveness and low computational complexity the latter is reduced further by a distributed approach based on Lagrangian relaxation and subgradient method. Experimental results demonstrate the effectiveness of the proposed scheduling strategies. For the further work, some heuristic approaches are required to be proposed to obtain the integer solutions after solving the quadratic-relaxed problem. Moreover, some genetic algorithms may also be applied under the distributed approach to reduce the computational complexity further.

REFERENCES


